general, a rough and ready policy is advocated; the principles of taxation which are advanced are so crude that they could not have found favor, even in the beginnings of the science. The principles of taxation advocated are such that if they were embodied in our revenue laws it is probable that they would seriously impair the industrial efficiency of the nation. In a volume which bears the subtitle "An account of the relation between private property and public welfare," one has a right to expect a better treatment of this important subject.

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THE FALLACY OF INDEX-NUMBERS.

A CONTINUOUS decline in prices since 1873 is supposed to be proven by the various tables of index-numbers which have been published. It is the examination of this method to which we shall give ourselves. It is a most remarkable fact that, although the method of index-numbers has been extensively used, especially in England, to determine the extent and direction of the general movement of prices, or its converse, the variations in the purchasing power of the money unit, still to my knowledge no one has yet clearly and conclusively exposed the worthlessness of this method of argumentation. Many valid objections have been urged to this table, or to that other, in regard to the reliability of the data on which it was based; other equally valid objections have been made to minor details in the application of the method; the trustworthiness of the method itself has been impugned on the ground that it does not give a quantitative value or "weight" to its percentages or ratios, and this objection, too, is valid and of great importance; but no one so far as I know has suspected that, by the method of index-numbers, quantity coefficients, whole or fractional, are covertly juggled into every single ratio determined by this method. The distortions caused by these unseen coefficients, which owe their origin to pure chance, are such as to entirely invalidate all the results arrived at by this method. It is time for the veneration of this fetich to cease.

To begin with, it will be well to insist that the word "average," when applied to price, demands the consideration of quantity; otherwise such average tacitly assumes the quantities to which the

prices refer to be equal. Thus, in a purchase of tops (for uniformity I shall always take the buyer's standpoint) when purchases are

2 tops @ 2c. = 4c.
1 top @ 8c. = 8c.
12c.

the average price is 4c.; at an average price of 5c. only one top could have been bought at each price. There has been the strangest carelessness on this point in investigations of price. The consideration of quantity is imperative, either in finding the average cost of the same commodity at different times, or of different commodities at the same time. Now an average price is the price which when multiplied by the total quantity will produce the total value. The average price per year of a commodity is to be found only in this way: whether these data can be furnished or not, is a question for the statistician to answer; if they cannot be found, they must be estimated; to neglect them would be fatal. The prices of the London *Economist* tables are given for selected dates for each year, and are therefore of little value. Though not very clearly, Dr. Soetbeer seems in his *Materialien* to imply that his average prices for the year are quantitative averages. Mr. Sauerbeck says of his prices (*Journal of the Statistical Society*, September 1886), "With but few exceptions the prices given are the average prices in each year, either those officially returned or the averages of the twelve quotations at the end of each month, partly received from private firms, partly collected from the *Economist* and other publications. Where a range of prices is given the mean has been taken."

To the extent that these prices are not quantitative averages they are unreliable.

When do the prices of two or more commodities taken together advance or decline, if the price of one or more commodities advance and the price of the others decline? We can say that the price of a commodity increases or diminishes when a fixed sum of money will buy a less or greater quantity; or that the price of a commodity increases or diminishes as the amount of money increases or diminishes which it takes to buy a fixed quantity of the commodity. In other words price is a ratio to which there are two terms; when either term is fixed it varies as the other. What is true of one commodity at different times is true of different commodities at the same time; provided that in the first case above, the quantities of the different commodities

are expressed in terms of the same unit of measure, pounds avoirdupois for instance. Thus no number which indicates the variations of price can possibly be an abstract number of "points," but is a concrete number, and in our monetary system it must be either dollars or units of measure.

Suppose that the prices of wheat and oats in three successive years were:

Year	Wheat	Oats	Total
1	$1.00	$0.50	$1.50
2	.85	.60	1.45
3	1.20	.35	1.55

Have the prices of wheat and oats taken together advanced or declined? The cost of one bushel of each of these grains shows that for this relation of quantity, prices of wheat and oats taken together have declined in the second year and have advanced in the third,—the comparison being in each case made with the first year. Now apply to these prices the method of index-numbers. Taking the prices of the first year as 100 and calculating the percentages,

Year	Wheat	Oats	Total
1	100	100	200
2	85	120	205
3	120	70	190

we take the sum of these percentages for each year but do not average the total. Professor Jevons is very explicit on this subject (*Serious Fall in the Value of Gold*, p. 6). "If a ton of bar iron costs £6, and a quarter of corn £3, there is no such relation or similarity as can warrant us in drawing an average between £6 and £3. If at a subsequent time a ton of iron costs £9, and a quarter of corn £3 12 s., there is again no average between these quantities. We may, however, say that iron has risen 50 per cent., or one-half; what was 100 has become 150. Corn has risen 20 per cent.; what was 100 has become 120. Now the ratios 100:150 and 100:120 are things of the same kind but of different amounts, between which we can strike an average. This average must not be the arithmetic, but the geometric average." It can be shown by the illustration given above that the similarity of the

percentages is a fiction and that no average of any kind between the totals can be constructed.

Taking the same prices of wheat and oats as before and reducing them to percentages, we here put the two tables side by side.

Year	Wheat	Oats	Total	Year	Wheat	Oats	Total
1	$1.00	$0.50	$1.50	1	100	100	200
2	.85	.60	1.45	2	85	120	205
3	1.20	.35	1.55	3	120	70	190

Here the index numbers of the second table show that with the same quantities (apparently) and the same prices as in the first table, prices advanced in the second and declined in the third year. Why do index-numbers here transform an advance into a decline, and *vice versa?* The next illustration will give the reason. Suppose the purchases to be two bushels of oats to every bushel of wheat; then with the same prices we have:

Year	Wheat (1 bu)	Oats (2 bu)		Total
1	$1.00	(2 × $0.50)	$1.00	$2.00
2	.85	(2 × .60)	1.20	2.05
3	1.20	(2 × .35)	.70	1.90

Except in being dollars these results coincide in every particular with those of the table of index-numbers above, and it now appears that when the quantity is fixed, price variations are registered by some number representing dollars. It is to this quantity relation that the index-numbers apply. Listen to Mr. Sauerbeck (*Journal*, 1886, p. 594) ". . . . they [index-numbers] take no notice of quantities and estimate all articles of equal importance." Not at all. What they actually do is to find the comparative cost of selected commodities in certain quantity relations which are determined for each commodity in the year taken as the base line; the quantity being the fraction or the multiple which its price in that year is of 100 times the money unit. In our illustration there are no prices over $1.00, therefore the quantity determinant is the money unit itself, 100 *cents*. When we assume that $1.00 = 100 we cannot at the same time assume that 50c. = 100 also, but we can, and in our illustration did, say covertly that 2 bu.

oats @ 50c. = $1.00 100 also. It is the arbitrary assumption that different and conflicting values are equal to 100 at the base line which makes the similarity of the percentages spoken of by Professor Jevons a fiction, and makes it rationally impossible even to add them together, to say nothing of averaging them. When we call them dollars, which they are in our table, we can add them together, for they are then similar things, viz., cost in dollars; but we can then average them only if the commodities are expressed in terms of the same unit of measure.

Let us now take our original illustration and test the prices there given by varying the quantity relations.

I.

Year	Wheat(1 bu)	Oats (1 bu)	Total
1	$1.00	$0.50	$1.50
2	.85	.60	1.45
3	1.20	.35	1.55

II.

Year	Wheat(1 bu)	Oats (2 bu)	Total
1	$1.00	$1.00	$2.00
2	.85	1.20	2.05
3	1.20	.70	1.90

III.

Year	Wheat(2 bu)	Oats (3 bu)	Total
1	$2.00	$1.50	$3.50
2	1.70	1.80	3.50
3	2.40	1.05	3.45

IV.

Year	Wheat(3 bu)	Oats (4 bu)	Total
1	$3.00	$2.00	$5.00
2	2.55	2.40	4.95
3	3.60	1.40	5.00

Comparing the cost for the second and third years with the first year in each case in these four tables, we find that, at the same moment of time and with the same prices, but with different relative quantities, the cost or price of wheat and oats taken together, in the second year declined in I, advanced in II, and remained stationary in III and that in the third year there is an advance in I, a decline in II and a stationary level in IV. Thus, for the different buyers above supposed the same prices at the same time mean to one an advance, to another a decline, and a stationary level to the third. And the purchasing power of the money unit of these different buyers, at the same time and with the same prices, increases, diminishes or remains stationary.

In tables I. to IV. we may say that the average proportion is four

bushels of wheat to seven bushels of oats, and applying this relation of quantity to our original prices,

Year	Wheat (4 bu)	Oats (7 bu)	Total
1	$4.00	$3.50	$7.50
2	3.40	4.20	7.60
3	4.80	2.45	7.25

we may say that on an average of the given quantity relations, cost has steadily advanced. This is the method of Mulhall in his *History of Prices since 1850*, which he calls the "trade level" method. He compares "the actual total of trade with the sums which the same volume of merchandise would have amounted to at previous periods, according to the prices then ruling." This method assumes that nations require certain fixed proportions of the principal commodities, which can be determined for a given year or averaged for a given series of years. Nothing can be further from the actual facts of the market. People do not buy herrings when sprats are cheap. A very large number of the most important commodities are interchangeable, and it is principally for this reason that if proportions, or rather fixed quantities of commodities in certain proportions, could be accurately determined for a given year or for a given series of years, the proportion of these quantities to one another would be quite arbitrary and misleading for comparison with other years.

Now, inasmuch as it is impossible to construct a table which will hold good of more than one of an infinite number of quantity relations, and inasmuch as in commerce quantities as well as proportions are constantly varying, it appears that tables and methods such as we have examined have no practical utility whatever, unless it be to furnish employment to some statistician in producing bogies to frighten "good honest folk" into the limbo called bimetallism in this country.

We conclude by reproducing the average prices of 1867 to 1877 of the commodities which enter into the table of Mr. Sauerbeck as 100, the base line, and add the quantities for which his results would hold good. There is a correction to be made in 13 of his articles to which he assigns one index-number each, but of which he gives two quotations of price. He calculates the percentage of each price separately and

BASE LINE PRICES OF MR. SAUERBECK'S TABLE.

No.	Description of Article	Denominations	Price	Quantities
1	Wheat, English	s. per quarter	54.5	1.835
2	" American	" "	56	1.786
3	Flour, town-made, white	" 280 lbs.	46	2.174
4	Barley	" quarter	39	2.564
5	Oats	" "	26	3.846
6	Maize	" "	32.5	3.077
7	Potatoes	" ton	117	.854
8	Rice	" cwt.	10	10
9	Beef, prime	d. per 8 lbs.	59	20.33
10	" middling	" "	50	24
11	mutton, prime	" "	63	19.044
12	" middling	" "	55	21.816
13	Pork	" "	52	23.076
14	Bacon	s. per cwt.	74	1.351
15	Butter	" "	125	.8
16	a Sugar, cane	" "	23	4.348
	b " beet	" "	24	4.166
17	" Java	" "	28.5	3.509
18	a Coffee, plantation	" "	87	1.149
	b " Brazil	" "	64	1.562
19	a Tea Congou	d. per lb.	11.25	106.668
	b " average import price	" "	17.25	69.564
20	Iron, pig	s. per ton	69	1.449
21	" bar	£ per "	8.25	.6061
22	a Copper, Chili bars	" "	75	.0666
	b " English cake	" "	81	.06172
23	Tin, Straits	" "	105	.04762
24	Lead	" "	20.5	.2439
25	Coals, London	s. per "	22	4.545
26	" average export price	" "	12.5	8
27	Cotton, medium upland	d. per lb	9	133.333
28	" fair Surat	" "	6.75	177.72
29	Flax	£ per ton	47	.1063
30	a Hemp, Manilla	" "	43	.1163
	b " St. Petersburg	" "	35	.1428
31	Jute	" "	19	.2632
32	a Wool, merino fleece	d. per lb	21.25	56.472
	b " Australian (grease)	" "	9.875	121.44
33	" Lincoln	" "	19.75	60.762
	" Southdown	" "	16	75.
34	Silk, Tsatlee	s. per "	23	4.348
	" Organsins	" "	42	2.381
35	a Hides, River Plate dry	d. per lb	9	133.333
	b " " salted	" "	7	171 48
36	Leather	" "	16	75
37	a Tallow	s. per cwt.	45	2.222
	b " town	" "	45	2.222
38	Oil, palm	£ per ton	39	.1282
39	" olive	" tun	50	.1
40	a " linseed	" ton	30	.1666
	b Seeds, linseeds	s. per qr.	60	.1666
41	Petroleum	d. per gal.	12.5	96
42	Soda, crystals	s. per ton	92	1.087
	Alkalin, export price	" "	185	.5406
43	Nitrate of soda	" cwt.	14	7.143
44	Indigo	" lb	7.25	13.79
45	a Timber, hewn	" load	60	1.666
	b " sawed or split	" "	54	1.852

enters the arithmetic mean of these two in his table. We calculate quantities for each as if they had each a distinct number in his table. Our column of figures gives real quantities expressed in terms of the unit of measure to which the price refers, and hence when prices are in pound sterling or pence these must be reduced to shillings or decimals of a shilling.

The relative quantities of the commodities of Mr. Sauerbeck's table, which we have above computed, and of which alone his results are valid — this purely accidental quantity relation, of the very existence of which he has not the remotest suspicion — we believe will be readily admitted, in view of its origin, to be entirely devoid of all authority; and with it vanishes the authority of the table itself. The results of the calculations of Professor Jevons, on the fluctuations in the purchasing power of gold, which are ceaselessly repeated in bimetallist literature, are likewise devoid of value, vitiated as they are by the fallacious reasoning of the the index-number method. The tables of Dr. Soetbeer and of the London *Economist* are similarly vitiated by the use of the method, the fallacy of which, I believe, has now been clearly and conclusively proven.

Proof of the varying purchasing power of gold, which has been sought to be established by means of these tables, would, if it could be obtained, be of the utmost importance, and would be welcomed by all who are interested in the question. For the reasons given above, however, it appears not alone that such a variation in the purchasing power of gold has not been proven, but that in the nature of things it cannot be proven.

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